Key

Math 4 Name ______________________________

2-2 Properties of Matrices Date______

Learning Goals:

- *I can determine if matrix multiplication is possible and determine the dimensions of the resulting matrix.*
- *I can multiply two matrices by hand.*
- *I can multiply two matrices using the graphing calculator.*
- *I can use matrix multiplication to solve real-life scenario problems.*
- *I can identify properties of matrices and compare them to those of the real numbers.*

I. Matrix Multiplication

A. There are two types of multiplication for matrices: **scalar multiplication** and **matrix multiplication**. In lesson 2-1 you learned about scalar multiplication, which is very straightforward as well as simple to perform. Multiplying a matrix times another matrix is a different story.....

Study the examples below and answer the questions that follow:

a. $\underbrace{\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}}_{1\times 4} \cdot \underbrace{\begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}}_{4\times 1} = \begin{bmatrix} 20 \end{bmatrix}$

b. $\underbrace{\begin{bmatrix} 3 & 2 & 1 & 4 \end{bmatrix}}_{1\times 4} \cdot \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}}_{4\times 2} = \begin{bmatrix} 10 & 10 \end{bmatrix}$

c. $\underbrace{\begin{bmatrix} 3 & -1 \end{bmatrix}}_{1\times 2} \cdot \underbrace{\begin{bmatrix} 5 & -2 \\ 0 & 3 \end{bmatrix}}_{2\times 2} = \begin{bmatrix} 15 & -9 \end{bmatrix}$

d. $\underbrace{\begin{bmatrix} -1 & 2 \\ 1 & -4 \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} 1 & -3 & 2 \\ -5 & 0 & -4 \end{bmatrix}}_{2\times 3} = \begin{bmatrix} -11 & 3 & -10 \\ 21 & -3 & 18 \end{bmatrix}$

➢ What do you notice about the resulting product matrices? What you end up with seems kind of random, perhaps. Not exactly. Go back and identify the orders for the two matrices being multiplied and their product.

These two matrices cannot be multiplied: $\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & -1 \end{bmatrix}}_{2\times 3} \cdot \underbrace{\begin{bmatrix} 2 & 4 & 5 \end{bmatrix}}_{1\times 3}$ "Error: Dimension error"

➢ Based on the matrices' orders, what must be true for them to be compatible for multiplying?

"Middle" #s must match. 1st matrix columns must be = to 2nd matrix rows.

➢ How can you predict what the order of the product matrix will be?

Product will have rows of 1st matrix & colums of 2nd matrix.

Put your new theory to work:

3 X 8 times a 8 X 5 = 3 X 5 10 X 3 times a 3 X 10 = 10 X 10

10 X 3 times a 10 X 3 = ____ X ____

Not possible! $\underbrace{10 \times 3 \cdot 10}_{\text{Have to Match}} \times 3$

B. Now that you have the whole compatibility idea down, let's see how the process of multiplying two matrices works. Revisit the multiplication problem below:
****Matrix multiplication actually involves not just multiplying, but adding as well!*

$$\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 20 \end{bmatrix}$$ Show how this process works for this example.

$$= \begin{bmatrix} 1\cdot 4 + 2\cdot 3 + 3\cdot 2 + 4\cdot 1 \end{bmatrix} = \begin{bmatrix} 4 + 6 + 6 + 4 \end{bmatrix}$$
$$= \boxed{\begin{bmatrix} 20 \end{bmatrix}}$$

$$\begin{bmatrix} 3 & 2 & 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 10 & 10 \end{bmatrix}$$ Show how this process works for this example.

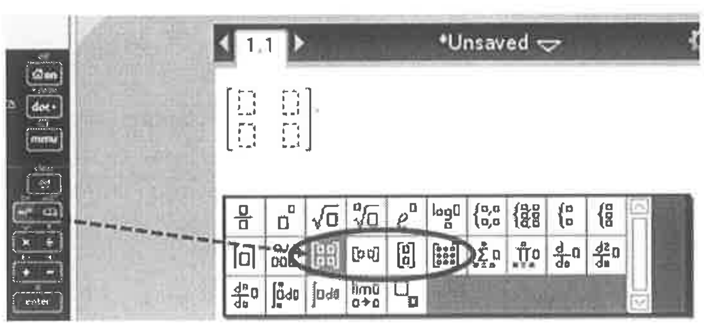
$$= \begin{bmatrix} 3\cdot 1 + 2\cdot 1 + 1\cdot 1 + 4\cdot 1 & 3\cdot 1 + 2\cdot 1 + 1\cdot 1 + 4\cdot 1 \end{bmatrix}$$
$$= \boxed{\begin{bmatrix} 10 & 10 \end{bmatrix}}$$

Now try this on your own: $\begin{bmatrix} 1 & -2 \\ -5 & 8 \end{bmatrix} \cdot \begin{bmatrix} -4 & 0 \\ -2 & 6 \end{bmatrix} =$

Show your work here:

$$\begin{bmatrix} -4 + 4 & 0 + -12 \\ 20 + -16 & 0 + 48 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & -12 \\ 4 & 48 \end{bmatrix}}$$

Check your final answer using your calculator. ☺



Just for fun . . . . Try checking your answer to the example by dividing your product matrix by one of the factors (in your calculator). What happens?

Error! Cannot divide matrices.

II. **Properties of Matrices**

There are several properties of the ***real numbers*** that you must remember in order to investigate the properties of matrices.

The first property is called the **commutative property of addition/multiplication**, which says $a+b=b+a$ and $a \cdot b = b \cdot a$. Show that this property is true (for both addition and multiplication) using the given a and b values:

1. $a=4,\ b=-6$

$4 + -6 = -2$ $(4)(-6) = -24$

$-6 + 4 = -2$ $(-6)(4) = -24$

2. $a=\frac{1}{2},\ b=\frac{2}{5}$

$\frac{1}{2} + \frac{2}{5} = \frac{9}{10}$

$\frac{2}{5} + \frac{1}{2} = \frac{9}{10}$

$\frac{1}{2} \cdot \frac{2}{5} = \frac{2}{10}$

$\frac{2}{5} \cdot \frac{1}{2} = \frac{2}{10}$ $\} \frac{1}{5}$

3. Is this property true for subtraction and/or division? Show yes or no using the a and b values from number (1).

$4 - -6 = 10$

$-6 - 4 = -10$ No!

$\frac{4}{-6} = -\frac{2}{3}$

$-6/4 = \frac{-3}{2}$ $\}$ No!

The second property is called the **associative property of addition/multiplication**, which says $(a+b)+c = a+(b+c)$ and $(a \cdot b) \cdot c = a \cdot (b \cdot c)$. Show that this property is true (for both addition and multiplication) using the given a, b, and c values.

4. $a=-2,\ b=5,\ c=4$

$(-2+5)+4 = 3+4 = 7$

$-2 + (5+4) = -2 + 9 = 7$

5. $a=\frac{1}{3},\ b=\frac{1}{4},\ c=\frac{5}{6}$

$\left(\frac{1}{3} \cdot \frac{1}{4}\right) \cdot \frac{5}{6} = \frac{1}{12} \cdot \frac{5}{6} = \frac{5}{72}$

$\frac{1}{3} \cdot \left(\frac{1}{4} \cdot \frac{5}{6}\right) = \frac{1}{3} \cdot \frac{5}{24} = \frac{5}{72}$

6. Is this property true for subtraction and/or division? Show yes or no using the a, b, and c values from number (4).

$(-2-5)-4 = -7-4 = -11$

$-2-(5-4) = -2-1 = -3$ No!

$(-2 \div 5) \div 4 = -\frac{1}{10}$

$-2 \div (5 \div 4) = \frac{-8}{5}$ No!

We are now going to explore whether or not these two properties hold for ***matrices***. Use the matrices below and your calculator to answer questions 7 – 14.

$$\text{Let } A = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 3 \\ -4 & 6 \end{bmatrix}, \text{ and } C = \begin{bmatrix} 7 & -3 \\ 8 & -7 \end{bmatrix}.$$

7. Does the **commutative property of addition** hold for matrices? That is, does $A+B=B+A$? Show yes or no below.

$\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ -4 & 6 \end{bmatrix} = \begin{bmatrix} 0 & 7 \\ -1 & 11 \end{bmatrix}$

$\begin{bmatrix} -2 & 3 \\ -4 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 7 \\ -1 & 11 \end{bmatrix}$

Yes!

8. Does the **commutative property of multiplication** hold for matrices? That is, does $A \cdot B = B \cdot A$? Show yes or no below.

$[A] \cdot [B] = \begin{bmatrix} -20 & 30 \\ -26 & 39 \end{bmatrix}$ $[B][A] = \begin{bmatrix} 5 & 7 \\ 10 & 14 \end{bmatrix}$

9. Does the **associative property of addition** hold for matrices? That is, does $(A+B)+C = A+(B+C)$? Show yes or no below.

$(A+B)+C = \begin{bmatrix} 7 & 4 \\ 7 & 4 \end{bmatrix}$ $A+(B+C) = \begin{bmatrix} 7 & 4 \\ 7 & 4 \end{bmatrix}$ Yes!

10. Does the **associative property of multiplication** hold for matrices? That is, does $(A \cdot B) \cdot C = A \cdot (B \cdot C)$? Show yes or no below.

$(A \cdot B) \cdot C = \begin{bmatrix} 100 & -150 \\ 130 & -195 \end{bmatrix} = A \cdot (B \cdot C)$

Now, we will go back to the ***real numbers*** and explore a few more properties. The first is the fact that, for numbers, there is both an **additive identity** and a **multiplicative identity.**

11. An **additive identity** is a number that you can add to *any number* without changing it. What is the additive identity? Think, $a +$ _0_ $= a$. The number that goes in the blank is the additive identity.

12. A **multiplicative identity** is a number that you can multiply by *any number* without changing it. What is the multiplicative identity? Think $a \cdot$ _1_ $= a$. The number that goes in the blank is the multiplicative identity.

13. Using matrix B, find the **additive identity matrix**. That is, fill in the blank with a matrix so that the following statement is true: $B + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = B$

14. Using matrix A, find the **multiplicative identity matrix**. That is, fill in the blank with a matrix so that the following statement is true: $A \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A$ This is a bit more challenging . . . .

The last property we will talk about is the **additive inverse** and the **multiplicative inverse**. An additive inverse is a number that, when you add it to a, the sum is the additive identity. A multiplicative inverse is a number that, when you multiply it by a, the product is the multiplicative identity.

What is the additive inverse of 7? _-7_ What is the multiplicative inverse of 7? _$\frac{1}{7}$_

What is the additive inverse of x? _$-x$_ What is the multiplicative inverse of x? _$\frac{1}{x}$_

We will discuss as a class additive and multiplicative inverses of matrices.

Summary of Properties

Properties	**Real Numbers** *Yes or No*	**Matrices** *Yes or No*
Commutative Property of Addition	Yes	Yes
Commutative Property of Subtraction	No	No
Commutative Property of Multiplication	Yes	$\boxed{\text{No}}$
Commutative Property of Division	No	No
Associative Property of Addition	Yes	Yes
Associative Property of Subtraction	No	No
Associative Property of Multiplication	Yes	Yes
Associative Property of Division	No	No
Properties	**Real Numbers** *What is it?*	**Matrices** *What is it?*
Additive Identity	0	$\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$
Multiplicative Identity *Identity matrices are always* Square 2x2, 3x3, etc.	1	$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$
Additive Inverse $5 + -5 = 0$	$-x$	Each entry has opposite sign
Multiplicative Inverse *Not all matrices have inverses. Ones that do are* Square *matrices.* $5 \cdot \frac{1}{5} = 1$	$\frac{1}{x}$	$[A]^{-1}$

Different from real #s!

$\begin{bmatrix}1 & 2\end{bmatrix} + \begin{bmatrix}-1 & -2\end{bmatrix} = \begin{bmatrix}0 & 0\end{bmatrix}$

NOTES:

Additive Inverse for Matrices:

$$\begin{bmatrix}2 & -1 & 3\\0 & .5 & -4\end{bmatrix} + \begin{bmatrix}-2 & 1 & -3\\0 & -0.5 & 4\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

Multiplicative Inverse for Matrices:

$$\begin{bmatrix}5 & 2 & 0\\5 & 3 & 0\\3 & 2 & 1\end{bmatrix}^{\boxed{-1}} = \begin{bmatrix}3/5 & -2/5 & 0\\-1 & 1 & 0\\1/5 & -4/5 & 1\end{bmatrix}$$

Complete: $\begin{bmatrix}5 & 2 & 0\\5 & 3 & 0\\3 & 2 & 1\end{bmatrix} * \begin{bmatrix}5 & 2 & 0\\5 & 3 & 0\\3 & 2 & 1\end{bmatrix}^{-1} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

Given $A = \begin{bmatrix}4 & 8\\3 & 6\end{bmatrix}$. Find A^{-1}. There isn't one!

Just because it's square doesn't mean it has an inverse.

Use the following matrices for problems 1 – 4.

$A=\begin{bmatrix}2 & -1\\ 0 & 3\end{bmatrix}$ 2×2 $B=\begin{bmatrix}1 & 0 & -6\\ -2 & 5 & 4\end{bmatrix}$ 2×3 $C=\begin{bmatrix}2 & -3\\ 5 & 1\end{bmatrix}$ 2×2 $D=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ 2×2

1. Perform the following operations by hand (*if possible*). Show your work.

$A*B$ $\begin{bmatrix}2 & -1\\ 0 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & -6\\ -2 & 5 & 4\end{bmatrix} = \begin{bmatrix}2+2 & 0+-5 & -12+-4\\ 0+-6 & 0+15 & 0+12\end{bmatrix} = \boxed{\begin{bmatrix}4 & -5 & -16\\ -6 & 15 & 12\end{bmatrix}}$

Identity!

$C*D$ $\begin{bmatrix}2 & -3\\ 5 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \boxed{\begin{bmatrix}2 & -3\\ 5 & 1\end{bmatrix}}$

$B*A$ Not possible! $2\times\underbrace{3\cdot 2}_{\text{No match}}\times 2$

2. Which matrix does not have an inverse? B
3. Which matrix is the multiplicative identity matrix? D
4. Matrix E (not shown above) is multiplied by matrix B so that $E \bullet B$ produces a 4 x 3 matrix. What are the dimensions of matrix E? 4 x 2 $\underline{4}\times\underline{2}\cdot 2\times 3 = 4\times 3$

*******Use your calculator for the remaining problems.***

5. Ralph and Trixie both run *ebay* businesses out of their parents' homes. They both need to buy some new DVDs and an *X-Box* 360 game to sell. The matrix below indicates the prices from two different stores for the desired items. They both want to shop at only one store, but they do not need to shop at the same store.

Cost (in dollars)

	Frozen	*Ted 2*	*Madden '16*
Best Deal	24.99	21.99	59.99
Bullseye	29.99	19.99	54.99

2x3

a. Ralph needs to buy 3 *Frozen* DVDs, 4 *Ted 2* DVDs, and 12 *Maddens*. Trixie needs to buy 5 *Frozen* DVDs, 1 *Ted 2* DVD, and 10 *Maddens*. Set up a matrix that represents this information. ***Be sure that the matrix can be multiplied by the cost matrix.***

b. Multiply the Cost matrix and your matrix from part a. Show all 3 matrices. Be sure to label the rows and columns all 3 matrices! Write your matrix equation below.

$$\begin{array}{c|ccc} & \text{Fr} & \text{Ted} & \text{Mad}\\ \hline \text{BD} & 24.99 & 21.99 & 59.99\\ \text{Bull} & 29.99 & 19.99 & 54.99\end{array} \cdot \begin{array}{c|cc} & \text{Ralph} & \text{Trixie}\\ \hline \text{Fr} & 3 & 5\\ \text{Ted} & 4 & 1\\ \text{Mad} & 12 & 10\end{array} = \begin{array}{c|cc} & \text{Ralph} & \text{Trixie}\\ \hline \text{BD} & 882.81 & 746.84\\ \text{Bull} & 829.81 & 719.84\end{array}$$

2×3 3×2

c. Identify the meaning of the entry in the first row, first column of your solution matrix. ***Be specific!***

Ralph would spend $882.81 at Best Deal.

d. At which store should each person shop? ***Explain.***

They should both shop at Bullseye. The bottom row of the product matrix has smaller numbers than the top row in both columns.

6. Solve the following equation for matrix *X*.

$$A = \begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix}.$$

$$2X + 3A = B$$

$2X = B - 3A$

$X = \dfrac{B-3A}{2} = \dfrac{\begin{bmatrix} 0 & 3 \\ 2 & 0 \\ -4 & -1 \end{bmatrix} - 3\begin{bmatrix} -2 & -1 \\ 1 & 0 \\ 3 & -4 \end{bmatrix}}{2} = \boxed{\begin{bmatrix} 3 & 3 \\ -0.5 & 0 \\ -6.5 & 5.5 \end{bmatrix}}$

7. Let matrices *A* and *B* be of orders 3×2 and 2×2 respectively. Answer the following questions and explain your reasoning.

a. Is it possible that $A = B$?

No. They have different order (diff dimensions).

b. Is $A + B$ defined?

Cannot add them since they have different orders.

c. Is *AB* defined? If so, is it possible that $AB = BA$?

A·B is defined $3 \times \underbrace{2 \cdot 2}_{\checkmark} \times 2$

So, $AB \neq BA$.

B·A is <u>not</u> defined $2 \times \underbrace{2 \cdot 3}_{\text{Not a Match}} \times 2$